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ABSTRACT

Generalizability or invariance procedures have been known for over three decades. Through the years, these procedures have not been widely discussed or employed. One reason for the lack of use is that most of the articles on invariance procedures have been mathematically oriented. The mathematical orientation of research articles and the lack of mathematical background on the part of educational researchers have inhibited the use of generalizability procedures. An increasing number of researchers is recognizing that significance testing does not inform the researcher regarding the replicability of results and are turning to cross-validation, jackknife, and bootstrap methods as part of their analyses. Bootstrap methods seem particularly useful. This paper presents two approaches to bootstrapping in the canonical case. The first method employs bootstrap estimates of the correlation matrix from which conventional canonical functions are then derived; these correlations can be computed using software such as that provided by C. E. Lunneborg (1987). The second approach employs bootstrap estimates of the canonical correlation coefficients; these coefficients are squared and the two extreme coefficients become the upper and lower bounds for the actual canonical coefficient. The data set used was composed of 21 college students who volunteered to take the Nelson Denny Reading Test. A 33-item list of references and seven data tables are included. (Author/TJH)

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TWO APPROACHES TO BOOTSTRAP ESTIMATION IN CANONICAL ANALYSIS

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ABSTRACT

Generalizability or invariance procedures have been known for over three decades. Through the years these procedures have not been widely discussed or employed. One reason for the lack of use is that the majority of the articles on invariance procedures have been mathematically oriented. The mathematical orientations of articles and the lack of mathematical background on the part of educational researchers have inhibited the use of generalizability procedures.

More and more researchers are recognizing that significance testing does not inform the researcher regarding the replicability of results and are turning to cross-validation, jackknife, and bootstrap methods as part of their analyses. Bootstrap methods seem particularly useful.

This paper presents two approaches to bootstraping in the canonical case. The first methods employs bootstrap estimates of the correlation matrix from which conventional canonical functions are then derived; these correlations can be computed using software such as that provided by Lunneborg (1987). The second approach employs bootstrap estimates of the canoncial correlation coefficients; these coefficients are squared and the two extreme coefficients become the upper and lower bounds for the actual canonical coefficient.

TWO APPROACHES TO BOOTSTRAP ESTIMATION IN CANONICAL ANALYSIS

The generalizability of research results have been under question for several years. Generally, a researcher wishes to generalize to the population from which a sample was drawn. But, results are open to skepticism if the sample may not be representative of the population or when the analytic methods employed require larger sample sizes than the research can utilize (Thompson, 1988). In addition, questions can be raised as to the reproducibility of the results because of the analytical methods used. But selected analytic methods can help resolve questions regarding the generalizability of results.

The paper discusses reasons why these generalizability analytic procedures are important. Also, three common generalizability procedures are reviewed. Concrete examples are provided using multivariate statistics to illustrate the uses of bootstrap.

Sample Representativeness

The first reason to use invariance procedures is to determine bounds for the results. Sample size is an important aspect to be considered for all parametric statistics, and even more so for multivariate statistics. Multiple dependent variables and their reliability increases a larger sample size for multivariate statistics. As variables, both independent and dependent, are

added to the equation, there is more question about the reliability of the results. A large sample is needed to off-set the question of reliability.

A small sample may not truly represent the population. Or, the sample may introduce questions about the reliability of the results due only to sample size. Thus, sample size is one design aspect that will determine if invariance procedures are important. Invariance procedures can aid in establishing some confidence and generalizability regarding obtained results.

When a small sample is used, the two invariance procedures recommended are bootstrap and jackknife. Bootstrap, which is a procedures that concatenates the data set, and jackknife, which divides the actual data set into multiple data sets, would be recommended. These procedures are recommended since the size of the data set is not significantly reduced by the methods.

The use of multiple invariance procedures in a given study is also recommended. Thompson (1984) stated that two procedures would provide with a range in with the results would probably fall within. A confidence level would be created. The use of two procedures is not only helpful when the researcher has a small sample size, but also when correlational statistics (i.e., factor analysis, canonical correlation, Pearson correlation, OVA procedures) are employed (Thompson, 1984). Statistical methods tend to capitalize on chance occurrences and would provide an inflated result (Nunnally, 1967; Thompson, 1984). The capitalization is greatest with the OVA techniques, since data is

lost often due to reducing interval data to nominal data. The invariance procedures would establish some bounds to guide interpretation.

Analytic Methods Requirements

Generalizability procedures are needed with some analytic methods more so than others. The multivariate statistical methods tend to capitalize on sampling error even more so than the univariate methods, since these methods are more sensitive to variation (Fornell, 1978). Multivariate statistics tend to be more sensitive to outliers, more so than univariate statistics. In addition to the type of analytic procedure the sample size needed for the analysis leads many researchers to use generalizability procedures. Most specifically, multivariate procedures tend to require larger sample sizes to provide more stability because of increased reliability variance and the sensitivity to variation (Balian, 1982; Katzell, 1951). Researchers may not have the luxury of a large sample and thus can rely on generalizability procedures to increase confidence in their interpretations.

Basis for Examples

The data set used in this paper is composed of 21 cases of college students who volunteered to take the Nelson Denny Reading Test. Vocabulary and comprehension scores were the dependent

variables. Additionally, reading rate (in words per minute), grade, and age were also included in the data set as the independent variables. Table 1 presents the data set.

INSERT TABLE 1 ABOUT HERE

Invariance Procedures

There are three common methods to estimate reproducibility or generalizability: (a) cross-validation, (b) jackknife, and (c) bootstrap. Each of these techniques introduce a different type of sampling logic.

The cross-validation procedure has been available to researchers prior to 1951. Mosier (1951) and Katzell (1951) discussed aspects of cross-validation. The cross-validation procedure calls for the researcher to split the actual data set into random samples. Gorsuch (1983, p. 334) recommends the cross-validation process be used in factor-analytic [or other correlational] studies to

> replicate the analysis by randomly splitting the sample in half, factoring both halves and identifying the factors that match across the two samples.

Thus, the particular analytical technique employed for the entire data set is redone on the two random samples. The weights derived from the analysis of the two random samples are crossed or, in the case of a factor analysis, the factors are compared. Synthetic

variables are created for the random sample using the actual weights and the crossed weights. The synthetic variables are compared using a Pearson product-moment correlation. If the variables are "highly" correlated, then the sample is said to be invariant. Thus, the results would be generalizable. Otherwise, the results would not be deemed generalizable.

The **jackknife** was first proposed by Quenouille (1949) and further refined by Tukey (1958). Jackknife is designed to "assess the accuracy or stability of an estimated parameter" (Ayabe, 1985, p. 445). The jackknife procedure is particularly useful for factor analysis (Pennell, 1972), and canonical correlation (Dempster, 1966). The jackknife procedure requires the actual data set to be divided into n groups of m size (Crask & Perreault, 1977). Then n numbers of analyses are run deleting one of the groups for each run. The same statistical analysis is used for the entire data set as well as every run of the groups. If the results are similar over the n iterations of the analysis then the results are considered stable.

Bootstrap is another estimator of bias and variance (Lunnenborg, 1985; Parr, 1983). Bootstrap provides a nonparametric inference which is broadly applicable across most parametric statistics (Diaconis & Efron, 1983; Efron, 1979, 1982; Efron & Gong, 1983; Lunneborg, 1985). Additionally, Efron (1980, 1982) demonstrated that the bootstrap can provide a more accurate depiction of the variability than jackknife.

The bootstrap procedures requires the researcher to copy the actual data set onto itself multiple times (Diaconis & Efron, 1983). Efron (1980) and Lunneborg (1985) recommended that the data set be copied between 100 to 1000 times onto itself. Then cases are randomly selected from this expanded data set to make samples. Each case has an equally good chance of being chosen for all samples. The researcher decides on the number of samples (usually dozens or hundreds) and runs the procedure on each sample. The results from the various runs are interpreted. A key statistic for the interpretation of the bootstrap procedure is the standard deviation of the estimates. The standard deviations provide the researcher with an indication of how stable the estimates are across the samples (Thompson, 1988).

> If variations in samples do not have much effect on the estimates, then these standard deviations will be small and the researcher can [have] more confidence in a the belief that the results will generalize (Thompson, 1988, p. 682).

Bootstrapping by Correlation Matrices

First, an original Pearson correlation was performed on the 21 original cases. Table 2 presents the correlation matrix from this original run. Also on the original data, a canonical correlation analysis was performed. The lambda for this run was .61497 or an effect size of 38.5 percent. Table 3 presents the

eigenvalue, canonical coefficient, and squared coefficient for both roots of the original analysis.

INSERT TABLES 2 AND 3 ABOUT HERE

Following the traditional analysis, the original data set was expanded from 21 to 2134 cases. With this new data set, ten Pearson correlation runs were performed with 21 cases randomly sampled from the expanded data set. A mean correlation matrix was derived from the ten correlation matrices using an MS-DOS based program developed by this author. The mean correlation matrix and standard deviations for the coefficients are presented in Table 4.

INSERT TABLE 4 ABOUT HERE

Notice that in Table 4 the standard deviations of the correlation coefficients are relatively small. This implies stability of the sampling and increases the confidence that the results will be generalizable.

The mean correlation matrix was used in the bootstrapped canonical analysis. This canonical analysis was then compared to the original analysis for similarity. Table 5 presents the bootstrapped canonical analysis.

INSERT TABLE 5 ABOUT HERE

Focusing on Table 5, notice that the squared canonical coefficients are extremely high for the bootstrapped canonical analysis than for the original analysis. Additionally, the eigenvalues are much higher for both roots in the bootstrapped analysis. The Wilks lambda is also extremely low (.00066). These results are in stark contrast with those found in the original analysis.

Investigating the structure coefficients, there are also differences. Table 6 present the structure coefficients for both roots for the original and bootstrapped data. The trends for the most important variable(s) are different between the two data. These differences indicate that the results are not invariant.

INSERT TABLE 6 ABOUT HERE

Bootstraping by Canonical Coefficients

As with the previous method, the original correlation analysis is needed and would be the primary focus of interpreting results. Bootstrapping by canonical coefficients estimates the reporducibility of the results.

Using the expanded data set, ten canonical correlation analyses were performed with 21 cases randomly selected for each run. The canonical coefficients and squared coefficients are used for this analysis and are listed in Table 7 The upper and lower bound for each root are created by the squared coefficients.

INSERT TABLE 7 ABOUT HERE

From Table 7, the upper and lower bounds for root 1 are .84937 and .36766, respectively. The bounds for root 2 are .35297 and .03031. Referring back to Table 3, the squared coefficients for root 1 is .28870 and for root 2 is .13572. Notice that root 1 does not fall within the bounds of the limits; it is actually below the lower limit. Also notice that root 2 does fall within the bounds created. Thus, root 2 is tending to be more reproducible than root 1. But, since the primary root (root 1) is not within the bounds, the results are said to be not invariant.

Summary

There are at least three generalizability procedures available to researchers. Of these, the most conservative is bootstrap. There are several method of achieving a bootstrapped result. Two methods were presented in this paper.

The importance of the use of generalizability procedures is to provide evidence of stability of the results. If the results are not generalizable within the data set, it is impossible for the results to be generalizable across the population.

Cross-validation, jackknife, and bootstrap only estimates the generalizability of results. But, these estimates increase the confidence in results for the researcher and reader. Thus, the credibility of the results are less likely to be questioned, which

is a by-product of a well thought out research design. The credibility of results is worth the time that it takes to perform a bootstrap or any other invariance procedures.

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Table 1
Data Set

ID	VOCAB	COMP	RATE	GRADE	AGE
1	37	28	195	15	20
2	65	58	381	14	17
3	62	62	210	14	22
4	35	42	262	13	18
5	43	34	247	13	19
6	66	44	184	13	29
7	32	34	134	15	20
8	57	38	215	14	20
9	33	42	172	13	16
10	36	30	215	14	30
11	44	48	220	13	17
12	38	38	310	14	18
13	46	48	215	14	21
14	64	50	262	13	18
15	48	46	247	13	18
16	90	56	247	15	18
17	75	54	262	14	18
18	41	44	262	13	18
19	48	56	411	13	18
20	75	50	429	13	18
21	45	52	391	13	18

Table 2
Original Correlation Matrix

	VOCAB	COMP	RATE	GRADE	AGE
VOCAB	1.0000				
COMP	0.6527 P=.000	1.0000			
RATE	0.2921 P=.030	0.4856 P=.001	1.0000		
GRADE	0.1187 P=.227	-.1909 P=.113	-.3577 P=.010	1.0000	
AGE	-.0616 P=.349	-.3412 P=.013	-.4054 P=.004	0.1218 P=.221	1.0000

Table 3

Canonical Correlation Coefficients for Original Data

Root No.	Eigenvalue	Canon Coef.	Sq. Coef.
1	.40588	.53731	.28870
2	.15704	.36841	.13572

Table 4

Mean Correlation Matrix from the Bootstrapped Matrices

	VOCAB	COMP	RATE	GRADE	AGE
VOCAB	1.0000				
COMP	0.6677 (.0711)	1.0000			
RATE	0.3699 (.2101)	0.5201 (.1371)	1.0000		
GRADE	-.3042 (.1959)	-.1984 (.1437)	-.5026 (.3864)	1.0000	
AGE	-.1899 (.1274)	-.4135 (.1964)	-.4382 (.0675)	0.2747 (.1756)	1.0000

Note. Standard deviations are in parentheses.

Table 5

Canonical Correlation Coefficients for Bootstrapped Data

Root No.	Eigenvalue	Canon Coef.	Sq. Coef.
1	846.27910	.99941	.99882
2	.78518	.66320	.43983

Table 6

Standardized Canonical Function Coefficients

Variables	Original Data		Bootstrapped Data	
	Root 1	Root 2	Root 1	Root2
VOCAB	-.5555	1.9740	-.2212	-35.0716
COMP	1.2697	-.3607	1.2210	35.0510
RATE	.5311	.9505	.9206	3.9424
GRADE	-.3241	.8621	.3760	14.3387
AGE	-.4900	.4151	-.2961	-18.3217

Table 7

Canonical Coefficients for
the Ten Bootstrapped Canonical Analyses

	ROOT 1		ROOT 2	
	CANON. COEF.	SQUARED COEF.	CANON. COEF.	SQUARED COEF.
1	.67145	.45085	.48263	.23293
2	.87761	.77020	.59412	.35297
3	.62530	.39099	.54366	.29556
4	.75982	.57733	.51093	.26105
5	.60635	.36766	.52104	.27149
6	.75174	.56512	.34805	.12114
7	.73068	.53389	.26839	.07203
8	.92161	.84937	.17411	.03031
9	.80577	.64926	.56328	.31729
10	.77425	.59987	.26781	.07172